

PCAC and the Deficit of Forward Muons in π^+ Production by Neutrinos

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Abstract

The K2K experiment, using a fine-grained detector in a neutrino beam of energy $< E > \sim 1.3$ GeV has observed two-track events that can be interpreted as a coherent reaction $\nu_\mu + \mathcal{N} \rightarrow \mu^- + \mathcal{N} + \pi^+$ ($\mathcal{N} = \text{C}^{12}$) or an incoherent process $\nu_\mu + (p, n) \rightarrow \mu^- + \pi^+ + (p, n)$, the final nucleon being unobserved. The data show a significant deficit of forward-going muons in the interval $Q^2 \lesssim 0.1 \text{ GeV}^2$, where a sizeable coherent signal is expected. We provide an explanation of this effect, using a PCAC formula that includes the effect of the non-vanishing muon mass. The deficit is caused by a destructive interference of the axial vector and pseudoscalar (pion-exchange) amplitudes. No such effect occurs in the neutral current channels such as $\nu_\mu + \mathcal{N} \rightarrow \nu_\mu + \pi^0 + \mathcal{N}$.

The K2K experiment has studied interactions of a low energy neutrino beam ($< E > \sim 1.3$ GeV) in a fine-grained detector, designed as the “near detector” of a long-base-line neutrino oscillation experiment [1, 2, 3]. Evidence has been found for two-track events, which can be interpreted as either $\nu_\mu + \mathcal{N} \rightarrow \mu^- + \mathcal{N} + \pi^+$ (coherent π^+ production on a nuclear target) or incoherent π^+ production $\nu_\mu + (p, n) \rightarrow \mu^- + \pi^+ + (p, n)$, where the final nucleon is unobserved. The data have been compared with simulations based on a model for coherent π^0 production [4], and a model for incoherent single pion production via nuclear resonances [5]. It is stated that in comparison with the simulations, the two-track data show “a significant deficit of forward-going muons” in the kinematic interval $Q^2 \lesssim 0.1 \text{ GeV}^2$, in which a sizeable coherent contribution is expected. In this Letter, we suggest a possible explanation of this effect.

As is well-known, neutrino scattering in the forward-scattering configuration is described by Adlers PCAC theorem [6]. For any inelastic charged current reaction $\nu_\mu + N \rightarrow \mu^- + F$, where F denotes an inelastic channel, the cross section, neglecting the muon mass, is

$$\left(\frac{d\sigma}{dx dy} \right)_{\text{PCAC}} = \frac{G^2 M E}{\pi^2} f_\pi^2 (1-y) \sigma(\pi^+ + N \rightarrow F) \Big|_{E_\pi = Ey} \quad (1)$$

where $x = Q^2/2MEy$ and $y = \nu/E$, ν being the energy transfer and E the neutrino energy. The pion decay constant has the value $f_\pi \approx 0.93 m_\pi$ and M denotes the nucleon mass. The extrapolation of the PCAC formula to non-forward angles is given by a slowly varying form-factor $[m_A^2/(m_A^2 + Q^2)]^2$, with $m_A \approx 1 \text{ GeV}$.

There is an important modification of Eq. (1) when the mass of the muon is taken into account. This modification can be found in a recent comment by Adler

[7], and can be expressed as a simple multiplicative correction factor

$$\mathcal{C} = \left(1 - \frac{1}{2} \frac{Q_{\min}^2}{Q^2 + m_\pi^2}\right)^2 + \frac{1}{4} y \frac{Q_{\min}^2 (Q^2 - Q_{\min}^2)}{(Q^2 + m_\pi^2)^2} \quad (2)$$

where

$$Q_{\min}^2 = m_l^2 \frac{y}{1-y}. \quad (3)$$

The range of the variable Q^2 is

$$Q_{\min}^2 \leq Q^2 \leq 2 M E y_{\max}, \quad (4)$$

where y lies between $y_{\min} = m_\pi/E$ and $y_{\max} = 1 - m_l/E$. Thus the corrected PCAC formula, valid for small angle scattering, is

$$\begin{aligned} \left(\frac{d\sigma}{dx dy}\right)_{\text{PCAC}, m_l \neq 0} &= \frac{G^2 M E}{\pi^2} f_\pi^2 (1-y) \sigma(\pi^+ + \mathcal{N} \rightarrow F) \Big|_{E_\pi = Ey} \\ &\cdot \mathcal{C} \theta(Q^2 - Q_{\min}^2) \theta(y - y_{\min}) \theta(y_{\max} - y) \end{aligned} \quad (5)$$

The physical interpretation of the correction factor \mathcal{C} is as follows: When the muon mass is not neglected, the reaction $\nu_\mu + \mathcal{N} \rightarrow \mu^- + F$ receives a contribution from the exchange of a charged pion between the lepton vertex $\nu \rightarrow \mu^-$ and the hadron vertex $\mathcal{N} \rightarrow F$. The coupling at the lepton vertex is $f_\pi m_l \bar{u}_\mu \gamma_5 u_\nu$, and the amplitude contains the characteristic pion propagator $(Q^2 + m_\pi^2)^{-1}$. This so-called pseudoscalar amplitude interferes with the remaining amplitude, which is free of the pion singularity, and which for $m_l = 0$ becomes proportional to $\langle F | \partial_\alpha A_\alpha | N \rangle$ in the forward scattering configuration. (For brevity we call this pole-free contribution the ‘‘axial’’ amplitude.) These two amplitudes interfere destructively. The destructive nature of the interference is visible in the first term of the correction factor \mathcal{C} , which reads $(1 - \frac{1}{2} \frac{Q_{\min}^2}{Q^2 + m_\pi^2})^2$. The two terms within the parentheses represent the axial and pseudoscalar amplitudes. The minus sign represents destructive interference. This, in our opinion, is the origin of the low- Q^2 deficit observed in the charged current π^+ production at low energies.

To see the impact of the correction factor \mathcal{C} we apply the modified PCAC formula (5) to the coherent process $\nu_\mu + \mathcal{N} \rightarrow \mu^- + \mathcal{N} + \pi^+$, using the same model for nucleon coherence used in describing coherent π^0 production [4]. This cross section has the form

$$\left(\frac{d\sigma^{\pi^+}}{dx dy d|t|}\right) = 2 \left(\frac{d\sigma^{\pi^0}}{dx dy d|t|}\right) \cdot \mathcal{C} \theta(Q^2 - Q_{\min}^2) \theta(y - y_{\min}) \theta(y_{\max} - y), \quad (6)$$

where $d\sigma^{\pi^0}/dx dy d|t|$ is given explicitly in Eq.(10) of Ref.[4].

Replacing x by $Q^2/(2MEy)$ and integrating over the variables t and y we obtain the Q^2 -distribution $d\sigma/dQ^2$. For the purpose of understanding the essential origin of the Q^2 suppression, it is enough to treat the pion-nucleon cross-section and the effects of nucleon absorption as constants, leaving the energy dependence of these quantities to a more complete calculation. The integration over the variable t then yields the approximate result

$$\left(\frac{d\sigma}{dy dQ^2}\right) \sim \frac{1-y}{y} \exp[-b|t|_{\min}] \mathcal{C}, \quad (7)$$

where $|t|_{\min}$ is given by

$$|t|_{\min} = 2 E^2 y^2 \left\{ 1 + \frac{Mx}{Ey} - \frac{m_\pi^2}{2 E^2 y^2} - \sqrt{1 + \frac{2Mx}{Ey}} \sqrt{1 - \frac{m_\pi^2}{E^2 y^2}} \right\}. \quad (8)$$

Integration over y produces the distribution $d\sigma/dQ^2$ shown in Fig. 1, which clearly exhibits the low- Q^2 suppression in the region $Q^2 < 0.1\text{GeV}^2$ which is experimentally observed [1].

Our explanation of the low- Q^2 deficit implies that the effect occurs only for charged current scattering, where the muon mass plays a role. The fact that the muon mass is of the same order as the pion mass appearing in the pion-propagator is important. The effect diminishes with increasing neutrino energy. We predict that the neutral current channels will show no low- Q^2 deficit. In this connection it is significant that the recent K2K measurement of $\nu_\mu + \mathcal{N} \rightarrow \mu^- + \mathcal{N} + \pi^0$ [8] finds that the angular distribution and the momentum spectrum of the π^0 are in good agreement with the Monte Carlo expectations [4, 5].

Even though the calculation of absolute coherent pion production entails considerable uncertainty, connected with rescattering and absorption effects within the nucleus, the PCAC effect discussed in this paper predicts a clear low Q^2 -suppression in the ratio of coherent charged current π^+ production to coherent neutral current π^0 production.

It is to be hoped that new high resolution detectors that are being developed in connection with long base line oscillation experiments will be able to test our explanation of the forward-muon deficit in an incisive way.

Acknowledgement: We are indebted to Dr. Yuichiro Kiyo, Volker Schulz and Georg Kreyerhoff for their kind assistance in the preparation of the manuscript.

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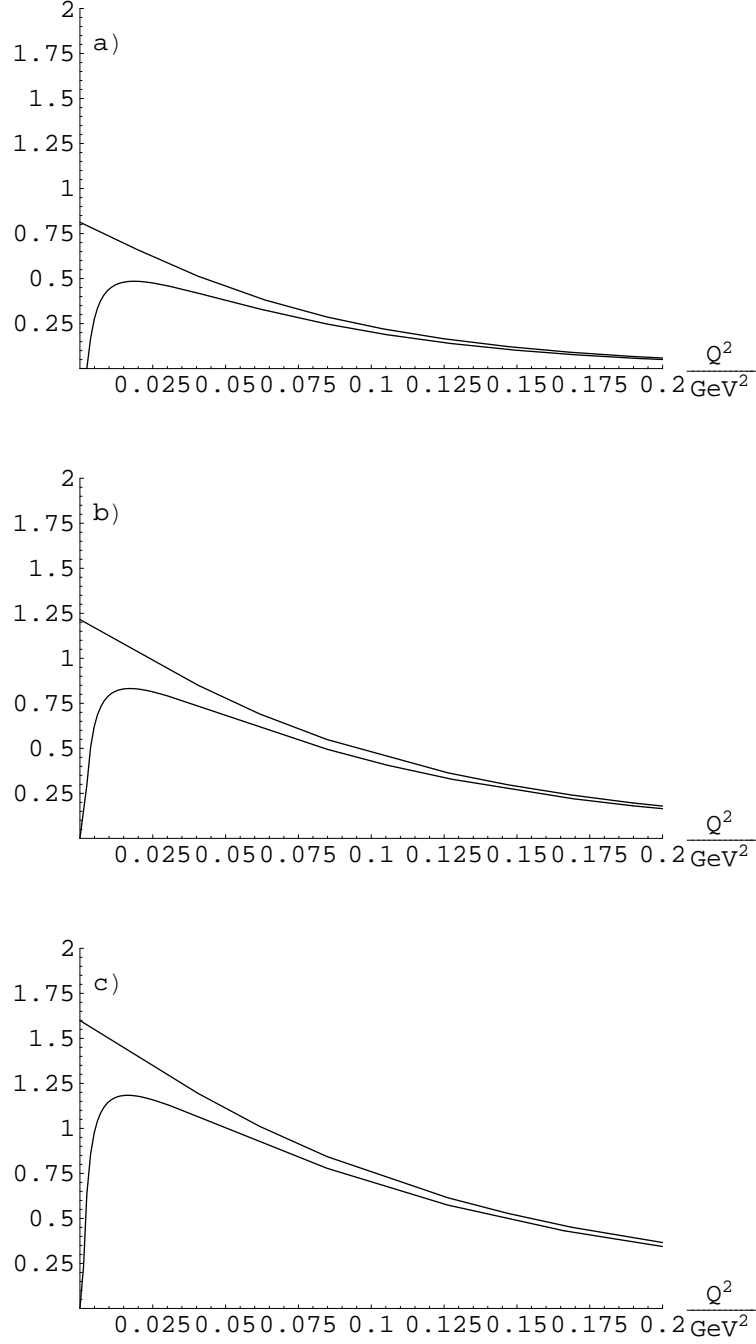


Figure 1: Effect of muon-mass correction on Q^2 -dependence of coherent π^+ production. Curves are derived from the simplified expression given in Eq. (8). The target nucleus is Carbon, and the neutrino energy is (a) 0.8 GeV, (b) 1.3 GeV and (c) 2.0 GeV. In each figure, the upper (lower) curve corresponds to the case $m_l = 0$ ($m_l \neq 0$)